International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

88087301

MATHEMATICS
STANDARD LEVEL
PAPER 1

Friday 7 November 2008 (afternoon)

1 hour 30 minutes

Candidate session number

| 0 | 0 | | | | | | | | |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1.   *[Maximum mark: 5]*

Consider the infinite geometric sequence $3, 3(0.9), 3(0.9)^2, 3(0.9)^3, \ldots$.

(a)   Write down the 10th term of the sequence. Do not simplify your answer.   *[1 mark]*

(b)   Find the sum of the infinite sequence.   *[4 marks]*

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

2.   *[Maximum mark: 6]*

A particle is moving with a constant velocity along line L. Its initial position is $A(6, -2, 10)$. After one second the particle has moved to $B(9, -6, 15)$.

(a)   (i)   Find the velocity vector, $\overrightarrow{AB}$.

   (ii)   Find the speed of the particle.   *[4 marks]*

(b)   Write down an equation of the line L.   *[2 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

3. *[Maximum mark: 7]*

Let $A = \begin{pmatrix} 1 & -2 \\ 3 & p \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ q & \frac{1}{2} \end{pmatrix}$.

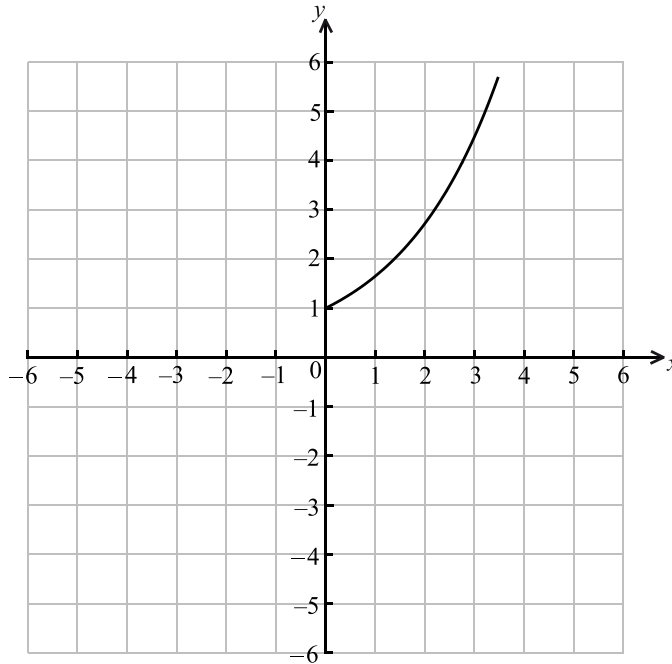   (a)   Find AB in terms of p and q.   *[2 marks]*

    (b)   Matrix B is the inverse of matrix A. Find the value of p and of q.   *[5 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

O411

4.   *[Maximum mark: 7]*

Let f be the function given by $f(x) = e^{0.5x}$, $0 \le x \le 3.5$. The diagram shows the graph of f.

(a)   On the same diagram, sketch the graph of f^{-1}.   *[3 marks]*

(b)   Write down the range of f^{-1}.   *[1 mark]*

(c)   Find $f^{-1}(x)$.   *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

5. *[Maximum mark: 6]*

Let A and B be independent events, where $P(A) = 0.6$ and $P(B) = x$.

(a)   Write down an expression for $P(A \cap B)$. *[1 mark]*

(b)   Given that $P(A \cup B) = 0.8$,

   (i)   find x;

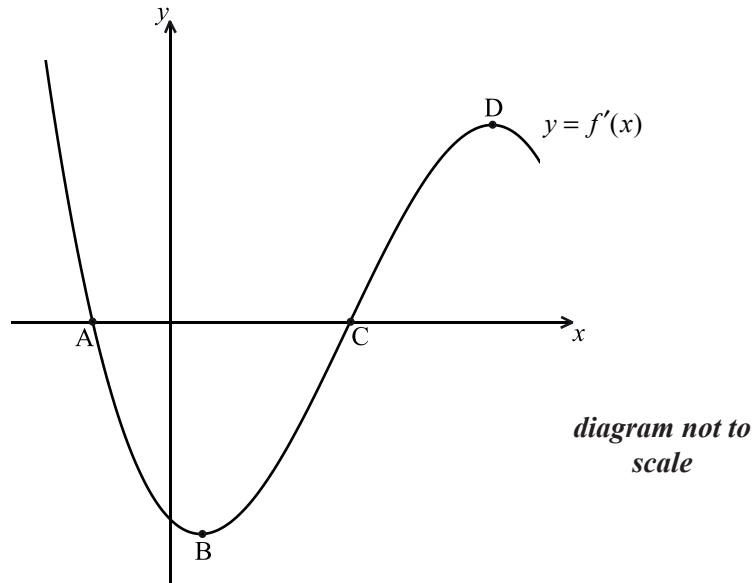   (ii)   find $P(A \cap B)$. *[4 marks]*

(c)   **Hence**, explain why A and B are **not** mutually exclusive. *[1 mark]*

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

0611

6. *[Maximum mark: 7]*

The diagram shows part of the graph of $y = f'(x)$. The *x*-intercepts are at points A and C. There is a minimum at B, and a maximum at D.

diagram not to scale

(a) (i) Write down the value of $f'(x)$ at C.

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same *x*-coordinate. *[3 marks]*

(b) Which of the points A, B, D corresponds to a maximum on the graph of f? *[1 mark]*

(c) Show that B corresponds to a point of inflexion on the graph of f. *[3 marks]*

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

O711

Turn over

7.   *[Maximum mark: 7]*

Let $f(x) = \sin^3 x + \cos^3 x \tan x$, $\dfrac{\pi}{2} < x < \pi$.

(a)   Show that $f(x) = \sin x$.   *[2 marks]*

(b)   Let $\sin x = \dfrac{2}{3}$. Show that $f(2x) = -\dfrac{4\sqrt{5}}{9}$.   *[5 marks]*

. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .
. . . .

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8.   *[Maximum mark: 13]*

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

Score on second die

	1	2	3	4	5	6
1	•	•	•	•	•	•
2	•	•	•	•	•	•
3	•	•	•	•	•	•
4	•	•	•	•	•	•
5	•	•	•	•	•	•
6	•	•	•	•	•	•

Score on first die

Let X be the sum of the scores on the two dice.

(a)   Find

    (i) $P(X = 6)$;

    (ii) $P(X > 6)$;

    (iii) $P(X = 7 \,|\, X > 5)$.   *[6 marks]*

(b)   Elena plays a game where she tosses two dice.

          If the sum is 6, she wins 3 points.
          If the sum is greater than 6, she wins 1 point.
          If the sum is less than 6, she **loses** k points.

    Find the value of k for which Elena's expected number of points is zero.   *[7 marks]*

Turn over

9.   *[Maximum mark: 16]*

The acceleration, $a \text{ ms}^{-2}$, of a particle at time t seconds is given by $a = 2t + \cos t$.

(a)   Find the acceleration of the particle at $t = 0$.   *[2 marks]*

(b)   Find the velocity, v, at time t, given that the initial velocity of the particle is 2 ms^{-1}.   *[5 marks]*

(c)   Find $\int_0^3 v \, dt$, giving your answer in the form $p - q \cos 3$.   *[7 marks]*

(d)   What information does the answer to part (c) give about the motion of the particle?   *[2 marks]*

1O11

10. *[Maximum mark: 16]*

Let $f(t) = a \cos b(t - c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.
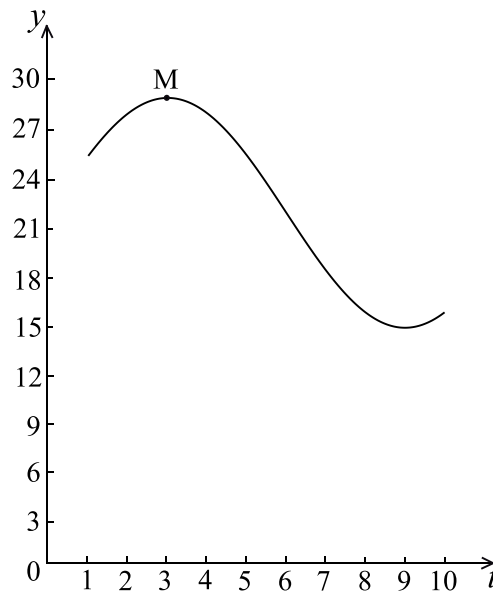
When $t = 3$, there is a maximum value of 29, at M.
When $t = 9$, there is a minimum value of 15.

(a) (i) Find the value of a.

    (ii) Show that $b = \dfrac{\pi}{6}$.

    (iii) Find the value of d.

    (iv) Write down a value for c. *[7 marks]*

The transformation P is given by a horizontal stretch of a scale factor of $\dfrac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$.

(b) Let M′ be the image of M under P. Find the coordinates of M′. *[2 marks]*

The graph of g is the image of the graph of f under P.

(c) Find $g(t)$ in the form $g(t) = 7 \cos B(t - C) + D$. *[4 marks]*

(d) Give a full geometric description of the transformation that maps the graph of g to the graph of f. *[3 marks]*